Homework 7 Oracle

MATH 220 Spring 2021

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Section 3.7

Problem 1 [FOR GRADE]

We wish to write the two sinusoidal terms as one.

$$
\begin{aligned}
3\cos 2t + 4\sin 2t &= R\cos(\omega_0 t - \delta) \\
&= R[\cos\omega_0 t\cos\delta + \sin\omega_0 t\sin\delta] \\
&= (R\cos\delta)\cos\omega_0 t + (R\sin\delta)\sin\omega_0 t
\end{aligned}
$$

Matching the coefficients, we obtain the following system of equations for ω_0, R, and δ.

$$
\begin{aligned}
R\cos\delta &= 3 \qquad (1) \\
\omega_0 &= 2 \qquad (2) \\
R\sin\delta &= 4 \qquad (3)
\end{aligned}
$$

Square both sides of the first and third equations

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$$R^2\cos^2\delta = 9$$
$$R^2\sin^2\delta = 16$$

and add their respective sides.

$$\begin{aligned} R^2\cos^2\delta + R^2\sin^2\delta &= 9+16 \\ R^2\left(\cos^2\delta + \sin^2\delta\right) &= 25 \\ R^2 &= 25 \\ R &= 5 \end{aligned}$$

Divide the respective sides of equations (1) and (3).

$$\frac{R\sin\delta}{R\cos\delta} = \frac{4}{3} \quad \to \quad \tan\delta = \frac{4}{3} \quad \to \quad \delta = \tan^{-1}\frac{4}{3}$$

Therefore,

$$3\cos 2t + 4\sin 2t = 5\cos\left(2t - \tan^{-1}\frac{4}{3}\right)$$

Problem 5

$$\begin{aligned} 20u'' + 400u' + 3920u &= 0 \\ 20r^2 + 400r + 3920 &= 0 \end{aligned}$$

then

$$r = -10 \pm 4\sqrt{6}i$$

We see that

$$\begin{aligned} u(t) =& C_1 e^{-10t}\cos(4\sqrt{6}t) + C_2 e^{-10t}\sin(4\sqrt{6}t) \\ u'(t) =& 4\sqrt{6}C_1 e^{-10t}\sin(4\sqrt{6}t) - 10C_1 e^{-10t}\cos(4\sqrt{6}t) \\ & + 4\sqrt{6}C_2 e^{-10t}\cos(4\sqrt{6}t) - 10C_2 e^{-10t}\sin(4\sqrt{6}t) \end{aligned}$$

By knowing that $u(0) = 2$ and $u'(0) = 2$, we find that $C_1 = 2$ and $C_2 = \frac{5}{\sqrt{6}}$. Finally,

$$u(t) = 2e^{-10t}\cos(4\sqrt{6}t) + \frac{5}{\sqrt{6}}e^{-10t}\sin(4\sqrt{6}t)$$

Therefore

$$\begin{aligned} \text{Quasi-frequency} &: 4\sqrt{6} \\ \text{Quasi-period} &: \frac{\pi}{2\sqrt{6}} \end{aligned}$$

Section 7.1

Problem 1 [FOR GRADE]

Let $u = x_1$.

$$x_1'' + 0.5x_1' + 2x_1 = 0$$

Finally, let $x_2 = x_1'$.

$$x_2' + 0.5x_2 + 2x_1 = 0$$

By making these substitutions, the original second-order ODE has become a system of first-order ODEs.

$$\begin{cases} x_1' = x_2 \\ x_2' = -2x_1 - 0.5x_2 \end{cases}$$

Problem 5

Let $u = x_1$.

$$x_1'' + p(t)x_1' + q(t)x_1 = g(t), \quad x_1(0) = u_0, \quad x_1'(0) = u_0'$$

Finally, let $x_2 = x_1'$.

$$x_2' + p(t)x_2 + q(t)x_1 = g(t), \quad x_1(0) = u_0, \quad x_2(0) = u_0'$$

By making these substitutions, the original initial value problem has become a system of first-order ODEs,

$$\begin{cases} x_1' = x_2 \\ x_2' = -q(t)x_1 - p(t)x_2 + g(t) \end{cases}$$

subject to the initial conditions,

$$x_1(0) = u_0 \quad \text{and} \quad x_2(0) = u_0'.$$

Section 7.2

Problem 4

A^T is the transpose of $A, \overline{A}$ is the complex conjugate of A, and $A^* = \overline{A}^T$ is the adjoint of A.

$$A^T = \begin{pmatrix} 3-2i & 2-i \\ 1+i & -2+3i \end{pmatrix}$$

$$\overline{A} = \begin{pmatrix} 3+2i & 1-i \\ 2+i & -2-3i \end{pmatrix}$$

$$A^* = \begin{pmatrix} 3+2i & 2+i \\ 1-i & -2-3i \end{pmatrix}$$

Problem 8 [FOR GRADE]

Start by calculating the determinant.

$$\det \begin{pmatrix} 1 & 4 \\ -2 & 3 \end{pmatrix} = (1)(3) - (4)(-2) = 11$$

Since it's not zero, an inverse for the given matrix exists.

$$\left(\begin{array}{rr|rr} 1 & 4 & 1 & 0 \\ -2 & 3 & 0 & 1 \end{array}\right)$$

The aim is to make the left side of the augmented matrix 1's and 0 's as the right side is now. Since the top left entry is 1 already, we move on to the bottom left entry. To make it zero, multiply both sides of the first row by 2 and add it to the second row.

$$\left(\begin{array}{cc|cc} 1 & 4 & 1 & 0 \\ 0 & 11 & 2 & 1 \end{array}\right)$$

To make the bottom right entry 1 , divide the bottom row by 11 .

$$\left(\begin{array}{cc|cc} 1 & 4 & 1 & 0 \\ 0 & 1 & \frac{2}{11} & \frac{1}{11} \end{array}\right)$$

To make the top right entry 0 , multiply the bottom row by -4 and add it to the first row.

$$\left(\begin{array}{cc|cc} 1 & 0 & \frac{3}{11} & -\frac{4}{11} \\ 0 & 1 & \frac{2}{11} & \frac{1}{11} \end{array}\right)$$

Therefore, the inverse matrix is

$$\begin{pmatrix} \frac{3}{11} & -\frac{4}{11} \\ \frac{2}{11} & \frac{1}{11} \end{pmatrix}.$$