Homework 6 Oracle

MATH 220 Spring 2021

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Section 3.3

Problem 7

$$y'' + 2y' + 2y = 0$$

Find that $r = \{-1 - i, -1 + i\}$. Then

$$y(t) = C_1 e^{-t} \cos(t) + C_2 e^{-t} \sin(t)$$

Problem 13 [FOR GRADE]

$$y'' - 2y' + 5y = 0, \quad y(\pi/2) = 0, \quad y'(\pi/2) = 2$$

Find that $r = \{1 + 2i, 1 - 2i\}$. Then

$$y(t) = C_1 e^{t} \cos(2t) + C_2 e^{t} \sin(2t)$$

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Let us apply the initial conditions

$$\begin{cases} y(\pi/2) = 0 \\ y'(\pi/2) = 2 \end{cases} \implies \begin{cases} C_1 = 0 \\ C_2 = -\frac{1}{e^{\pi/2}} \end{cases}$$

Finally,

$$y(t) = -\frac{1}{e^{\pi/2}} e^t \sin(2t)$$

Problem 17

$$5u'' + 2u' + 7u = 0, \quad u(0) = 2, \quad u'(0) = 1$$

Part (a)

Find that $r = \{-\frac{1}{5} - i\frac{\sqrt{34}}{5}, -\frac{1}{5} + i\frac{\sqrt{34}}{5}\}$. Then

$$\begin{cases} u(0) = 2 \\ u'(0) = 1 \end{cases} \implies \begin{cases} C_1 = 2 \\ C_2 = \frac{7}{\sqrt{34}} \end{cases}$$

Finally,

$$u(t) = 2e^{-t/5} \cos\left(\frac{\sqrt{34}t}{5}\right) + \frac{7}{\sqrt{34}} e^{-t/5} \sin\left(\frac{\sqrt{34}t}{5}\right)$$

Part (b)

$$T \approx 14.512$$

Problem 19

$$
\begin{aligned}
W\left(e^{\lambda t}\cos\mu t, e^{\lambda t}\sin\mu t\right) &= \begin{vmatrix} e^{\lambda t}\cos\mu t & e^{\lambda t}\sin\mu t \\ \frac{d}{dt}\left(e^{\lambda t}\cos\mu t\right) & \frac{d}{dt}\left(e^{\lambda t}\sin\mu t\right) \end{vmatrix} \\
&= \begin{vmatrix} e^{\lambda t}\cos\mu t & e^{\lambda t}\sin\mu t \\ \lambda e^{\lambda t}\cos\mu t - \mu e^{\lambda t}\sin\mu t & \lambda e^{\lambda t}\sin\mu t + \mu e^{\lambda t}\cos\mu t \end{vmatrix} \\
&= e^{\lambda t}\cos\mu t\left(\lambda e^{\lambda t}\sin\mu t + \mu e^{\lambda t}\cos\mu t\right) - e^{\lambda t}\sin\mu t\left(\lambda e^{\lambda t}\cos\mu t - \mu e^{\lambda t}\sin\mu t\right) \\
&= \lambda e^{2\lambda t}\cos\mu t\sin\mu t + \mu e^{2\lambda t}\cos^2\mu t\lambda e^{2\lambda t}\cos\mu t\sin\mu t + \mu e^{2\lambda t}\sin^2\mu t \\
&= \mu e^{2\lambda t}\left(\cos^2\mu t + \sin^2\mu t\right) \\
&= \mu e^{2\lambda t}
\end{aligned}
$$

Section 3.4

Problem 1

$$y'' - 2y' + y = 0$$

Then

$$r^2 - 2r + 1 = 0$$

So $r = 1$, finally

$$y(t) = C_1 e^t + C_2 t e^t$$

Problem 8 [FOR GRADE]

$$2y'' + 2y' + y = 0$$

Then

$$2r^2 + 2r + 1 = 0$$

So $r = \{-1/2 - 1/2i, -1/2 + 1/2i\}$. Finally,

$$y(t) = C_1 e^{-t/2} \cos\left(\frac{1}{2}t\right) + C_2 e^{-t/2} \sin\left(\frac{1}{2}t\right)$$

Problem 11

$$y'' + 4y' + 4y = 0, \quad y(-1) = 2, \quad y'(-1) = 1$$

Then $r = \{-2\}$. So

$$y(t) = C_1 e^{-2t} + C_2 t e^{-2t}$$

Taking the derivative

$$y'(t) = C_1 e^{-2t} - 2C_1 t e^{-2t} - 2C_2 e^{-2t}$$

Applying the initial conditions

$$\begin{cases} y(-1) = -C_1 e^2 + C_2 e^2 = 2 \\ y'(-1) = C_1 e^2 + 2C_1 e^2 - 2C_2 e^2 = 1 \end{cases} \implies \begin{cases} C_1 = \frac{5}{e^2} \\ C_2 = \frac{7}{e^2} \end{cases}$$

Therefore

$$y(t) = \frac{5}{e^2} t e^{-2t} + \frac{7}{e^2} e^{-2t}$$

As $t \to \infty$, $y \to 0$.

Problem 18 [FOR GRADE]

$$t^2 y'' - 4ty' + 6y = 0, \quad t > 0; \quad y_1(t) = t^2$$

Let us apply the method of reduction Let the general solution be of the form

$$y(t) = v(t) y_1(t) = t^2 v(t)$$

Let us find the derivatives

$$
\begin{aligned}
y_2'(t) &= t^2 v'(t) + 2tv(t) \\
y_2''(t) &= t^2 v''(t) + 2tv'(t) + 2tv'(t) + 2v(t)
\end{aligned}
$$

Let us substitute the above into our original ODE

$$
t^2(t^2 v''(t) + 2tv'(t) + 2tv'(t) + 2v(t)) - 4t(t^2 v'(t) + 2tv(t)) + 6(t^2 v(t)) = 0
$$

Simplifying yields

$$
\begin{aligned}
t^4 v''(t) + 3t^3 v'(t) + t^2 v(t) - 4t^3 v'(t) - 8t^2 v(t) + 6t^2 v(t) &= 0 \\
t^4 v''(t) &= 0
\end{aligned}
$$

Then

$$
v''(t) = 0
$$

Integrate both sides

$$
\begin{aligned}
v'(t) &= C_1 \\
v(t) &= C_1 t + C_2
\end{aligned}
$$

Finally, the general solution is

$$
y(t) = (C_1 t + C_2)t^2 = C_1 t^3 + C_2 t^2 = C_1 y_1(t) + C_2 y_2(t)
$$

and the second solution is $y_2(t) = t^3$.