Homework 3 Oracle

MATH 220 Spring 2021

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67; 12021 H.E.

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Section 2.3

Problem 1 [FOR GRADE]

We model the tank problem the following way of

$$\frac{dx}{dt} = R_{in} - R_{out}$$

Then

$$\frac{dx}{dt} = -\frac{2x}{200} = -\frac{x}{100}$$

We know the original concentration of $1 g/L$, then apply the separable equations method

$$\int \frac{100}{x} dx = \int -1 dt$$

Solving it yields

$$x = 200e^{-\frac{t}{100}}$$

We need to find the time that will elapse before the concentration of dye in the tank reaches 1%. So

$$\frac{x(t)}{x(0)} = 0.01 \implies 0.01 = e^{-\frac{t}{100}}$$

Solving for t returns that $t = 100 \ln 100$ min ≈ 460.5 min.

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Problem 4

Part (a)

So our kinetic and potential energies are equal. Then

$$mgh = \frac{1}{2}mv^2 \implies v = \sqrt{2gh}$$

Part (b)

Recall

$$\frac{dv}{dt} = A(h)\frac{dh}{dt} \quad \text{and} \quad \frac{dv}{dt} = av$$

So then because constant α is contracting, the change is negative

$$A(h)\frac{dh}{dt} = -\alpha a\sqrt{2gh}$$

Part (c)

Recall $A(r) = \pi r^2$. Let $h = 3$ and $\alpha = 0.6$. Then solving for radius of 1m and the circular outlet radius of 0.1m, we get an equation

$$A(1)\frac{dh}{dt} = -(0.6) \times A(0.1)\sqrt{2gh} \implies \pi\frac{dh}{dt} = -0.006\pi\sqrt{2gh} \implies \frac{dh}{dt} = -0.006\sqrt{2gh}$$

Solving for t yields ≈ 130.41.

Problem 9 [FOR GRADE]

Part (a)

See that

$$\frac{Q(5730)}{Q_0} = 0.5 \implies \frac{Q_0 e^{-r(5730)}}{Q_0} = 0.5 \implies e^{-r(5730)} = 0.5$$

Solve for r to get $r = 0.00012097 yr^{-1}$

Part (b)

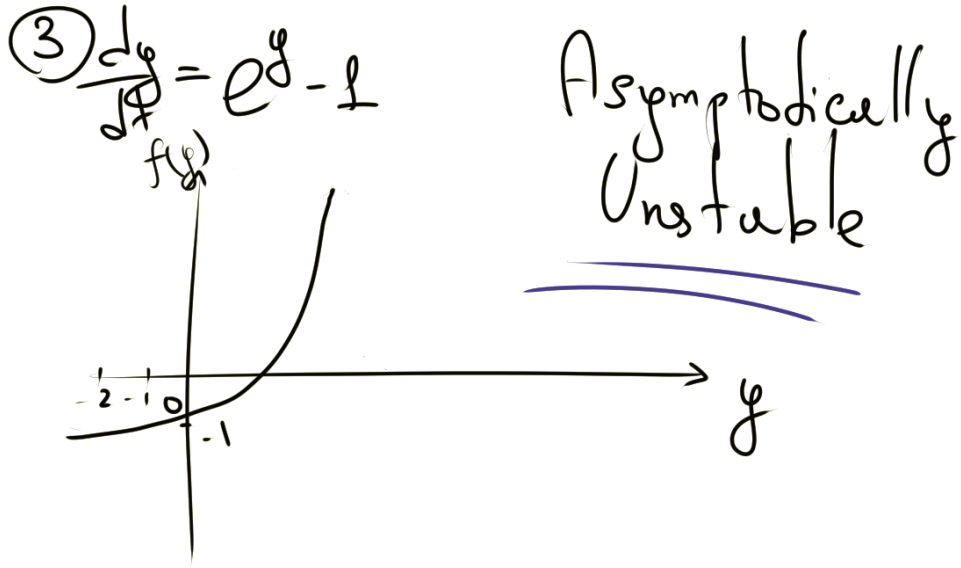
Trivially, it would be $Q_0 \exp(-0.00012097t)$, where t is measured in years.

Part (c)

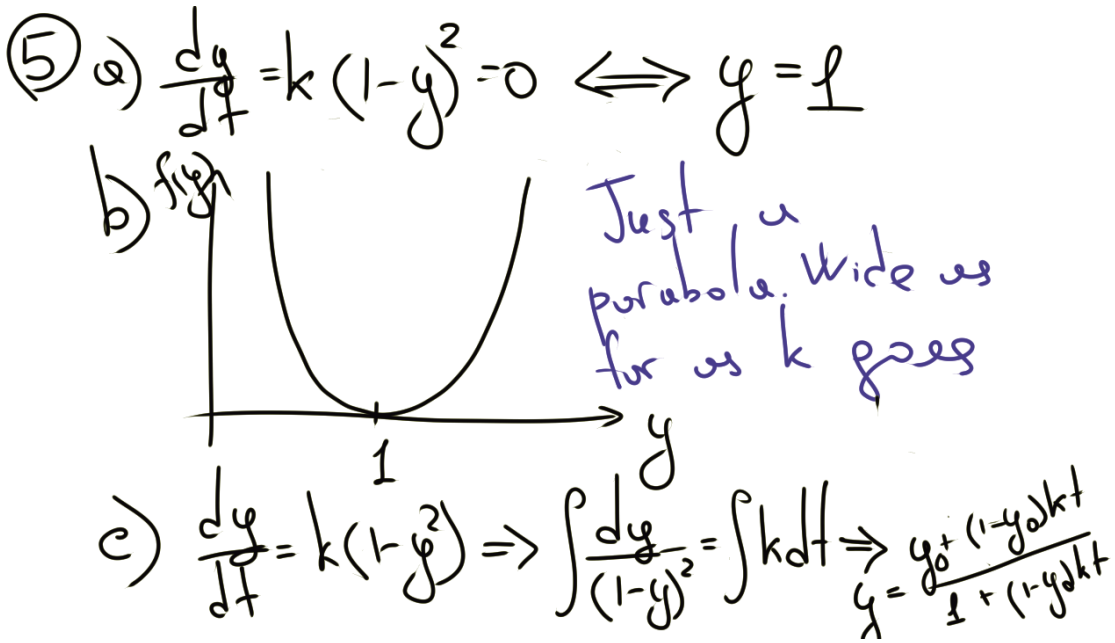
Solve $e^{-rt} = 0.2$ to get $13,305\text{yr}$.

Section 2.5

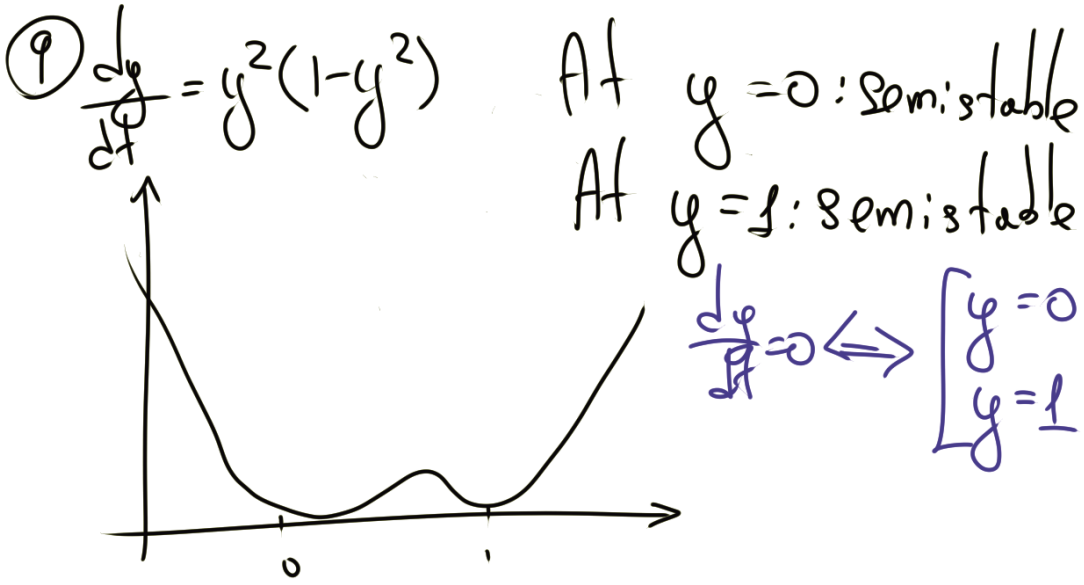
Problem 3



Problem 5



Problem 9 [FOR GRADE]



Problem 13

$$y_{1,2} = \frac{K + T \pm \sqrt{K^2 - KT + T^3}}{3}$$