Homework 10 Oracle

MATH 220 Spring 2021

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Hi, this is the last oracle published (homework 11 was published first because it had material used in the final exam). Instead of detailed solutions, we shall limit ourselves to just answers in this oracle. Thanks

Section 7.8

Problem 1

Statement

$$\mathbf{x}' = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix} \mathbf{x}$$

Answer

$$\mathbf{x} = c_1 \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^t + c_2 \left(\begin{pmatrix} 2 \\ 1 \end{pmatrix} t e^t + \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^t \right)$$

Problem 3

Statement

$$\mathbf{x}' = \begin{pmatrix} -\frac{3}{2} & 1 \\ -\frac{1}{4} & -\frac{1}{2} \end{pmatrix} \mathbf{x}$$

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Answer

$$x = c_1 \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{-t} + c_2 \left(\begin{pmatrix} 2 \\ 1 \end{pmatrix} te^{-t} + \begin{pmatrix} 0 \\ 2 \end{pmatrix} e^{-t} \right)$$

Problem 8 (this is graded)

Statement

$$x' = \begin{pmatrix} 3 & 9 \\ -1 & -3 \end{pmatrix} x, \quad x(0) = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$$

Answer

$$x = 2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} + 14 \begin{pmatrix} 3 \\ -1 \end{pmatrix} t$$

Problem 9(a)

Statement

$$x' = \begin{pmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 3 & 6 & 2 \end{pmatrix} x, \quad x(0) = \begin{pmatrix} -1 \\ 2 \\ -30 \end{pmatrix}$$

Answer

$$x = \begin{pmatrix} -1 \\ 2 \\ -33 \end{pmatrix} e^t + 4 \begin{pmatrix} 0 \\ 1 \\ -6 \end{pmatrix} te^t + 3 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} e^{2t}$$

Problem 10(a)

Statement

$$x' = \begin{pmatrix} -\frac{5}{2} & 1 & 1 \\ 1 & -\frac{5}{2} & 1 \\ 1 & 1 & -\frac{5}{2} \end{pmatrix} x, \quad x(0) = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$$

Answer

$$x = \frac{4}{3} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} e^{-t/2} + \frac{1}{3} \begin{pmatrix} 2 \\ 5 \\ -7 \end{pmatrix} e^{-7t/2}$$

Section 6.1

Problem 5

Statement

Find the Laplace transform of $f(t) = \cos(at)$, where a is a real constant. Recall that

$$\cosh(bt) = \frac{1}{2} \left(e^{bt} + e^{-bt} \right) \text{ and } \sinh(bt) = \frac{1}{2} \left(e^{bt} - e^{-bt} \right)$$

Answer

$$F(s) = \frac{s}{s^2 + a^2}, \quad s > 0$$

Problem 7 (this is graded)

Statement

In each of Problems 6 through 7, use the linearity of the Laplace transform to find the Laplace transform of the given function; a and b are real constants.

$$f(t) = \sinh(bt)$$

Answer

$$F(s) = \frac{b}{s^2 - b^2}, \quad s > |b|$$

Problem 12

Statement

In each of Problems 12 through 15, use integration by parts to find the Laplace transform of the given function; n is a positive integer and a is a real constant

$$f(t) = te^{at}$$

Answer

$$F(s) = \frac{1}{(s-a)^2}, \quad s > a$$