Project 3 - Mass of a Ravioli With Triple Integrals
MATH 147, Calculus III, Honors

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1 Model of the Ravioli Filling

1.1 Filling

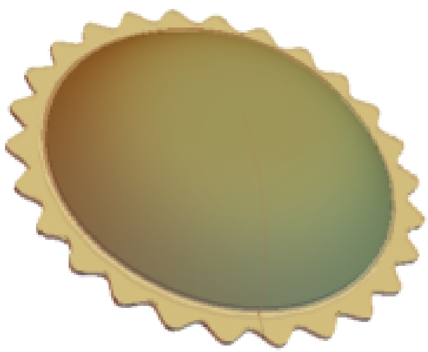

Figure 1: Ravioli

Figure 2: Ravioli Cross-Section

Figure 3: Ravioli Filling

1.2 Equations for Base

$$T_{Base}(r,t,h) = \begin{cases} x(r,t,z) = f(t) r \cos t \\ x(r,t,z) = f(t) r \sin t \quad & (r,t,z) \in D \\ x(r,t,z) = z \end{cases}$$

where

$$\begin{aligned} f(t) &= 1.5 \sin(26t) + 35 \\ D &= \{(r,t,h) : 0 \le r \le 1, 0 \le t \le 2\pi, 0 \le z \le 1.5\} \end{aligned}$$

1.3 Equations for Top

$$T_{top}(r,t,h) = \begin{cases} \begin{cases} x(r,t,h) = 35r \cos t \\ y(r,t,h) = 35r \sin t \\ z(r,t,h) = 12.5 \cos(2r^2) + h \end{cases} & if \quad (r,t,h) \in D_1 \\ \\ \begin{cases} x(r,t,h) = g(t) \cos t \\ y(r,t,h) = g(t) \sin t \\ z(r,t,h) = h \end{cases} & if \quad (r,t,h) \in D_2 \end{cases}$$

where

$$\begin{aligned} g(t) &= 35 + 12.5(r - 0.88) \sin(26t) \\ D_1 &= \{(r,t,h) : 0 \le r \le 0.88, 0 \le t \le 2\pi, 0 \le h \le 1.5\} \\ D_2 &= \{(r,t,h) : 0.88 \le r \le 1, 0 \le t \le 2\pi, 0 \le h \le 1.5\} \end{aligned}$$

1.4 Equations for the Filling

The Filling and RegionFunction commands in Mathematica™ are used to provide a solid fill

$$T_{filling}(r,t,h) = \begin{cases} x(r,t) = 35r\cos t \\ x(r,t) = 35r\sin t \\ x(r,t) = 12.5\cos(2r^2) \end{cases}$$

where

$$D = \{(r,t,h) : 0 \le r \le 0.88, 0 \le t \le 2\pi\}$$

2 Computing the Total Mass of the Ravioli

In this section we will compute the total mass of one ravioli. We will assume that the density of mass is given below.

$$\text{Ravioli dough:} \quad \delta_d = 1.14 \times 10^{-3}\frac{g}{mm^3}$$

$$\text{Ravioli filling:} \quad \delta_f = 1.01 \times 10^{-3}\frac{g}{mm^3}$$

2.1 Computing The Mass For the Bottom of the Ravioli

$$\text{Mass of base } = \iiint_{Base} \delta_d \, dV = \int_0^{2\pi}\int_0^1\int_0^{1.5} (1.5\sin(26t)^2 r \, dz \, dr \, dt \approx 6.58689$$

2.2 Computing The Mass For the Top of the Ravioli

$$\text{Mass of top } = \iiint_{Top} \delta_d \, dV = \int_0^{2\pi} dt \int_0^{0.88} dr \int_0^{1.5} 1.395r \, dz + \int_0^{2\pi} dt \int_{0.88}^{1} dr \int_0^{1.5} (1.395r) \, dz \approx 6.58808$$

2.3 Computing the Mass for the Ravioli Filling

$$\text{Mass of filling } = \iiint_{Filling} \delta_f \, dV = 0.00101 \times 4 \times \int_0^{30.8} dx \int_0^{\sqrt{30.8^2 - x^2}} dy \int_0^{12.5\cos\left(\left(\frac{x}{35}\right)^2 + \left(\frac{y}{35}\right)^2\right)} dz \approx 24.28747$$

3 Estimating a Necessary Amount of Ravioli

3.1 Computing the Mass For One Ravioli

6.58808 (mass of top) + 24.28747 (mass of filling) + 6.58689 (mass of base) = 37.46244 grams

3.2 Estimating the Amount of Ravioli Packages to Feed 11 Students

Each of the 11 students will eat 5 raviolis, while each package contains 500 grams of ravioli. This means that there are approximately 13 raviolis per package and that 55 raviolis will be eaten amongst the 11 students. Dividing 55 raviolis by 13 raviolis per package yields approximately 4.23 packages. Rounding up to 5 packages will ensure that every student receives 5 raviolis, in which there are 10 left over.

4 Conclusions

4.1 Finding the mass

In order to calculate the mass of the ravioli, it was necessary to find the mass of the top of the ravioli and the internal filling (the mass of the bottom was provided). In order to find the top, two triple integrals configured in cylindrical coordinates were used, as the formula of the top was split into two sections and was parameterized as such. In order to account in the shift of regions from cartesian to cylindrical, the determinate of the Jacobian matrix of the two formulas had to be determined for each integral. Once the Jacobians had been found, the volume of the top was computed, which was then multiplied by the density to find the mass. This was done similarly for the filling, however a triple integral in cartesian coordinates was utilized. Bounds in the XY plane were found with the equation of a circle with a radius of 30.8mm. The Z direction relied on the cartesian converted Z equation that parameterized the top of the ravioli. Challenges involved converting this equation to cartesian and finding a way in which the filling integral could be evaluated by Mathematica™. These integrals could also be solved by using a change of variables.

4.2 Communications and Group Work Plan

- Group Communications: Garrett Crossnoe
- File Management: Paul Negedu
- Mathematical Notation and Consistency of the Project: Alfred Fontes and Sagindyk Urazayev
- Document Preparation: Alfred Fontes and Sagindyk Urazayev